Wireless Communication with Multiple Antennas

Fundamentals of Antenna Arrays

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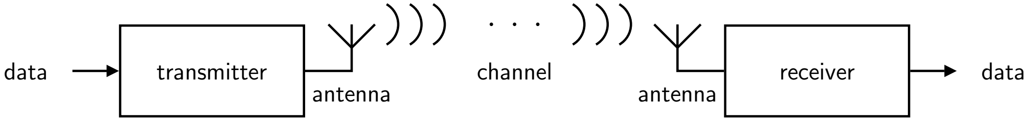
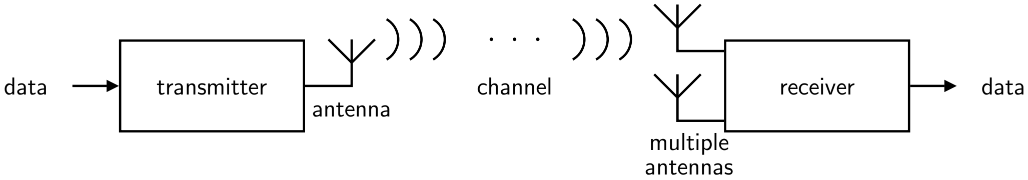
April 7, 2023


data
transmitter
antenna
channel
antenna
receiver
data


data
transmitter
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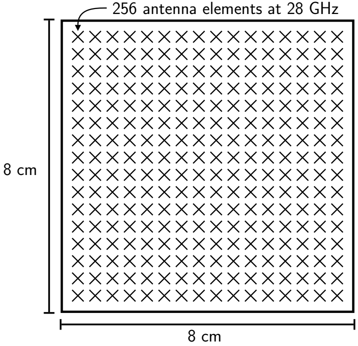
multiple antennas

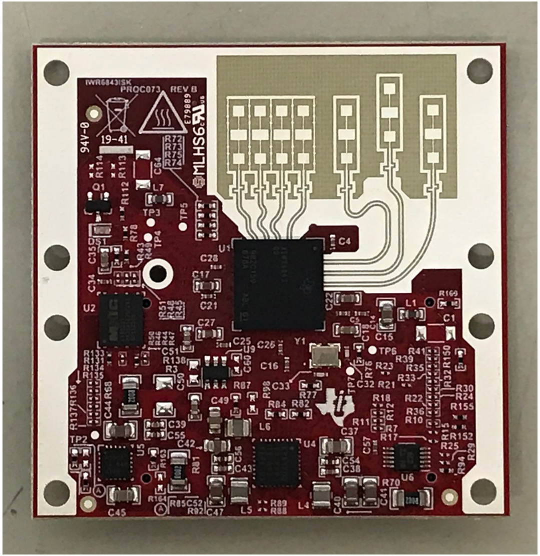
Wi-Fi Router

https://www.amazon.com/WiFi-6-Router-Gigabit-Wireless/dp/B08H8ZLKKK

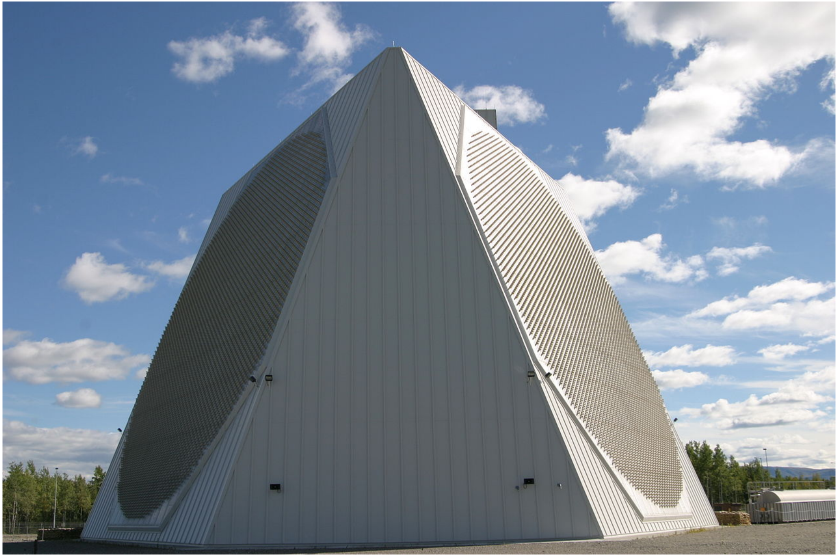
Anokiwave 28 GHz Transceiver for 5G Cellular Systems



Texas Instruments Millimeter Wave Radar



U.S. Early Warning Radar in Alaska



Very Large Array Radio Observatory in New Mexico

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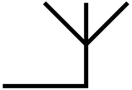
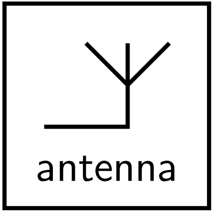


antenna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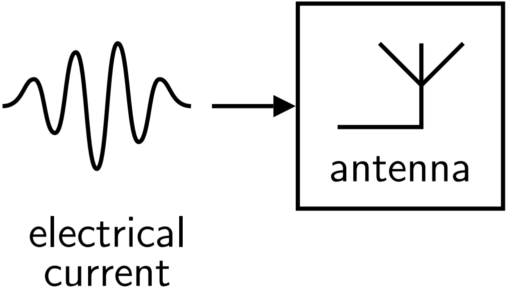
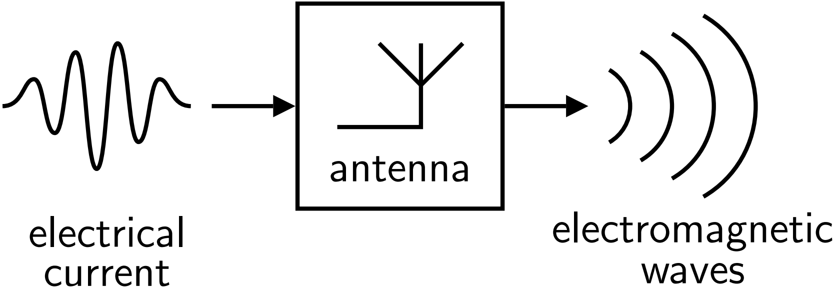
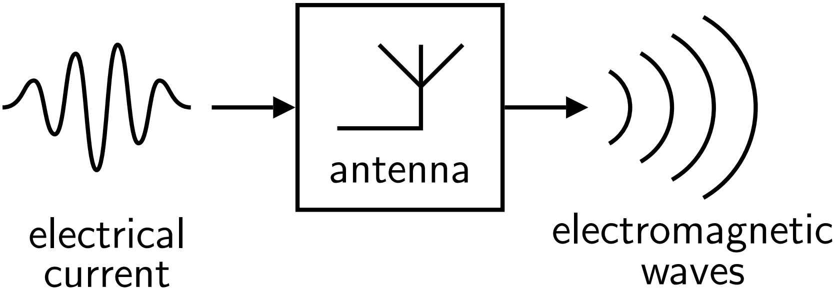

electrical
current
antenna


antenna
electrical current
electromagnetic waves

Antennas exhibit transmit-receive reciprocity.



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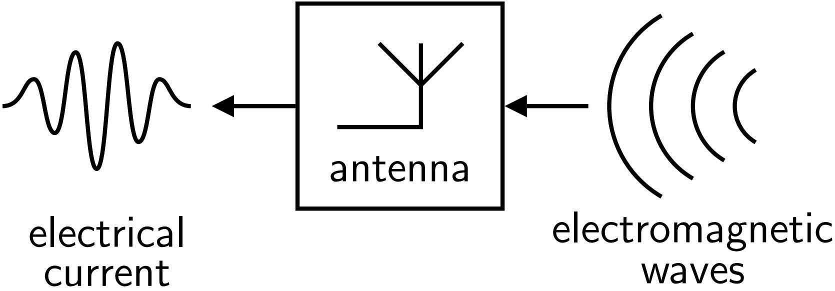
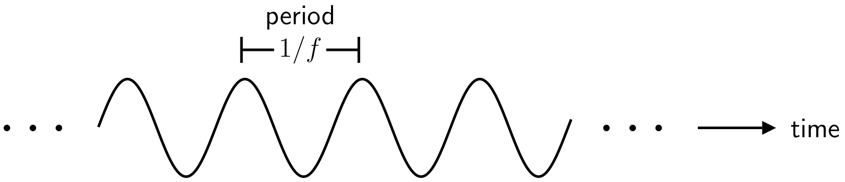


Electromagnetic waves propagate at the speed of light, $c \approx 3 \times 10^8$ meters/sec.



$y(t) = \sin(2\pi f t)$

a sinusoidal wave

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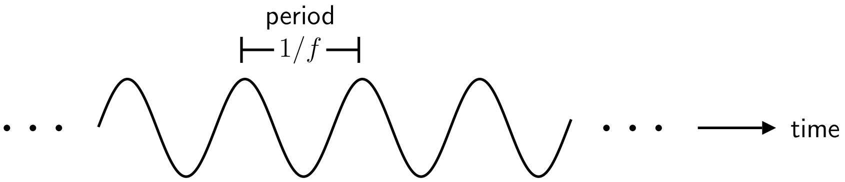

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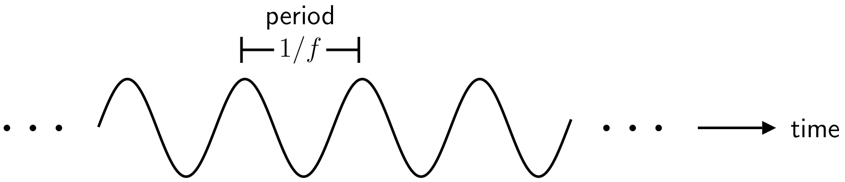

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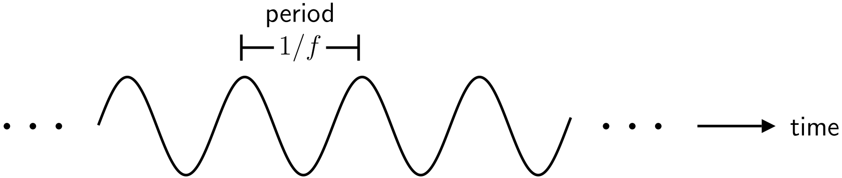

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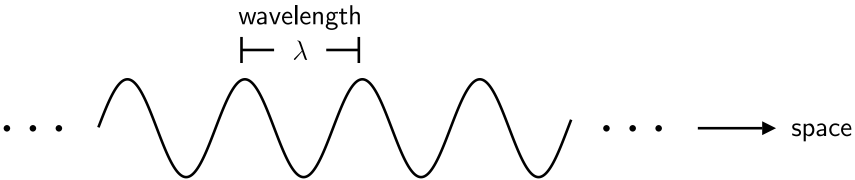

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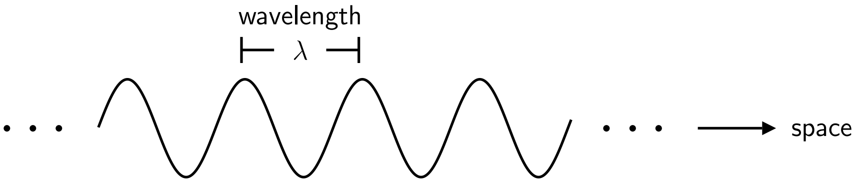
$$\underbrace{\mathrm{c}}_{\text{meters/sec}} = \underbrace{\lambda}_{\text{meters/cycle}} \cdot \underbrace{f}_{\text{cycles/sec}} \tag{1}$$



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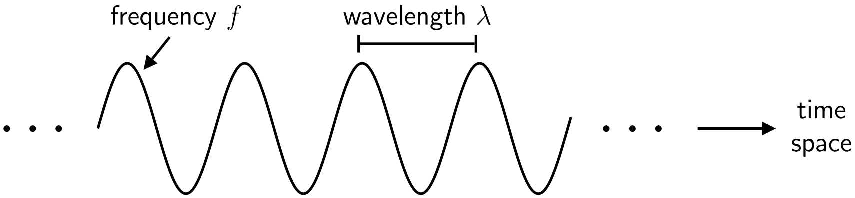
2.4 GHz Wi-Fi: frequency $f = 2.4$ GHz, wavelength $\lambda = 12.5$ cm.

5 GHz Wi-Fi: frequency $f = 5$ GHz, wavelength $\lambda = 6$ cm.



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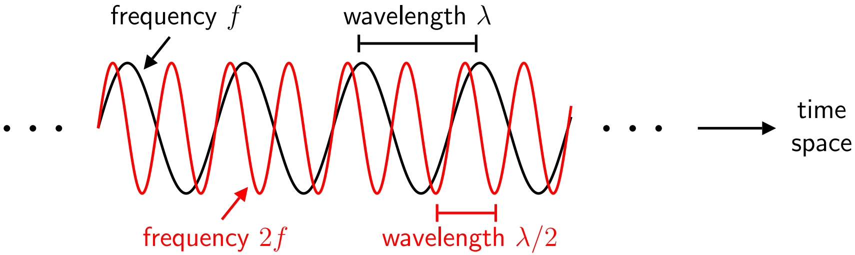

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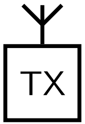
- but useful tool for studying antennas

Consider a transmitter with an isotropic antenna.



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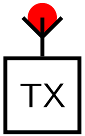

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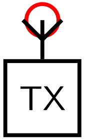

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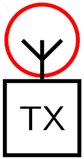

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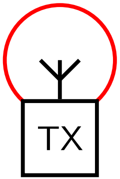

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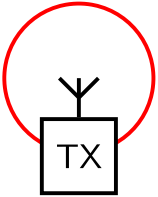

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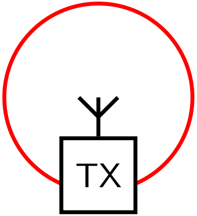

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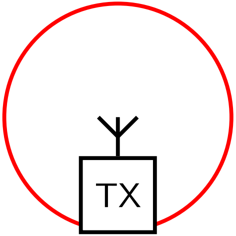

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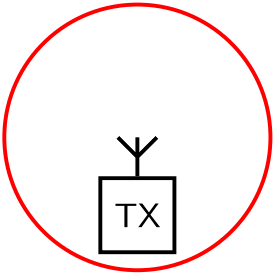

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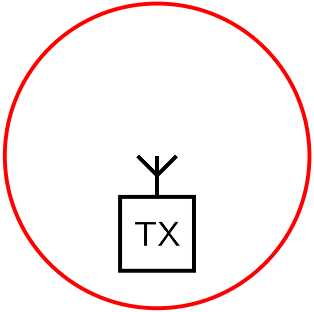

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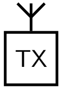

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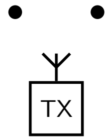

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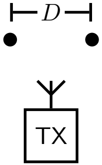

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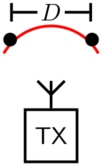

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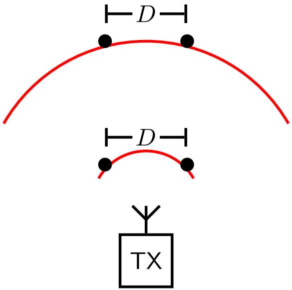

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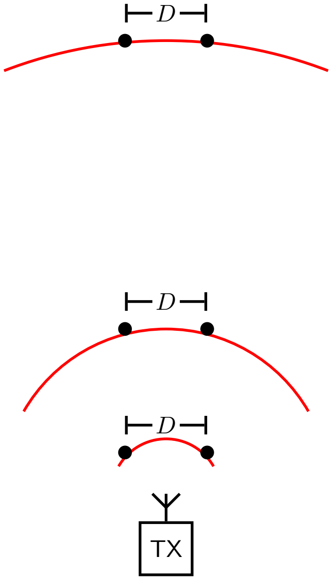

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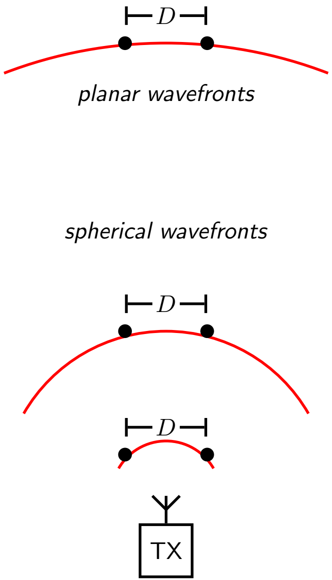

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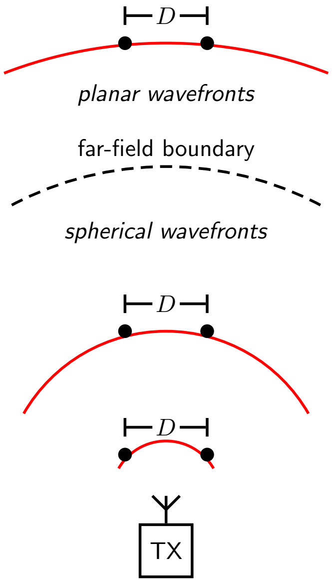
Far away, waves appear planar beyond the Rayleigh distance

$$\text{Rayleigh distance} = 2D^2/\lambda \tag{2}$$

when received by a real-world antenna whose largest dimension is D meters.



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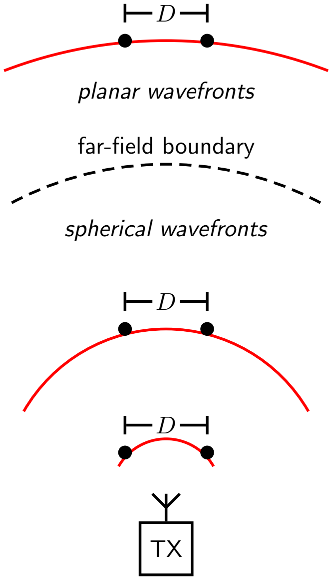
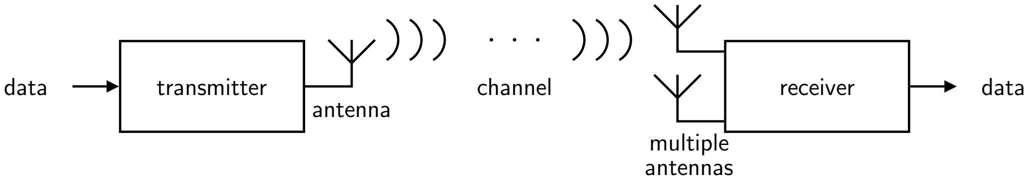
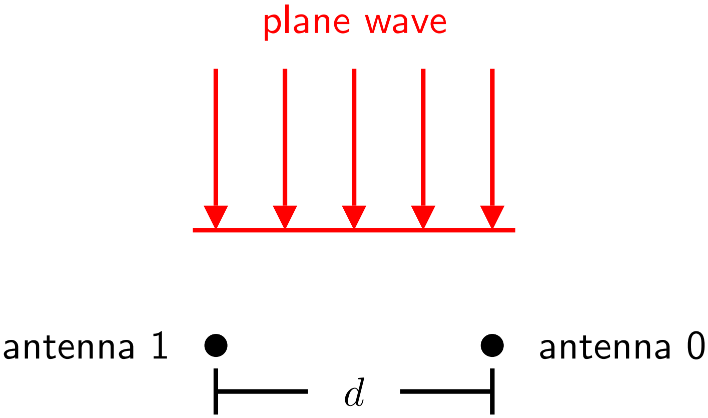
Also called the "far-field" or "Fraunhofer" distance.



Let's consider a communication system where a single-antenna transmitter communicates with a receiver with two antennas, located in the far-field.



Since we're in the far-field, a planar wavefront impinges the receive antennas.



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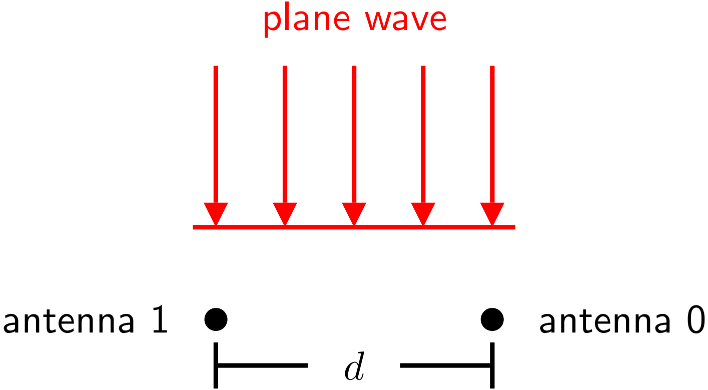

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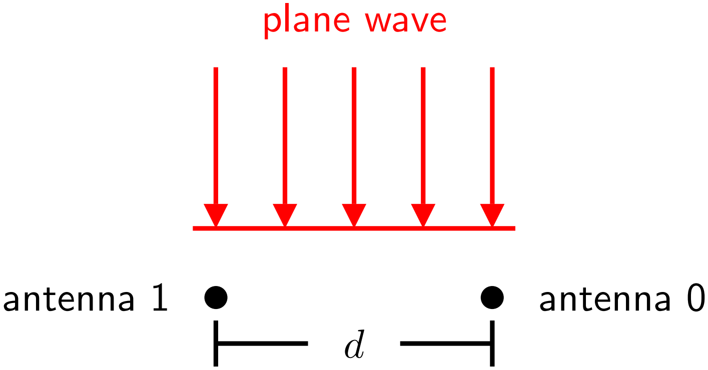

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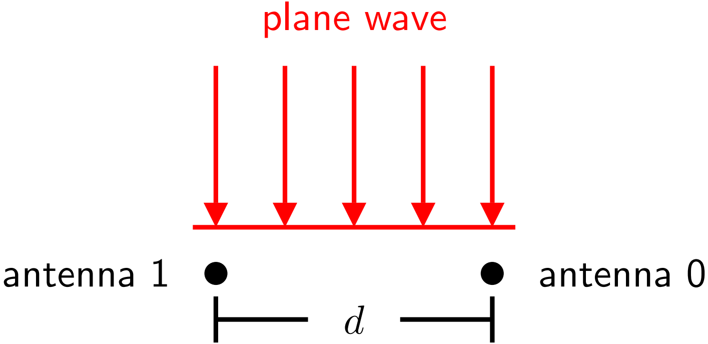

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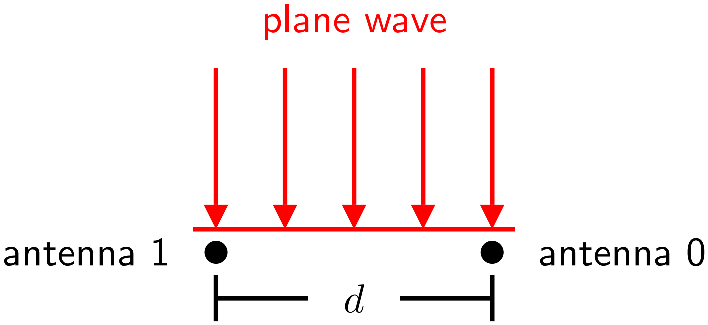

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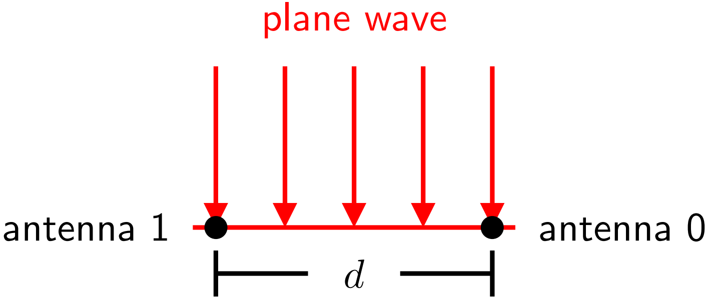

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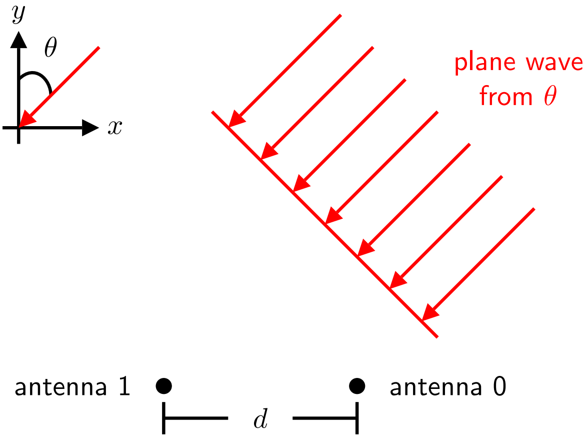
What about when antennas are not perpendicular to the direction of propagation?



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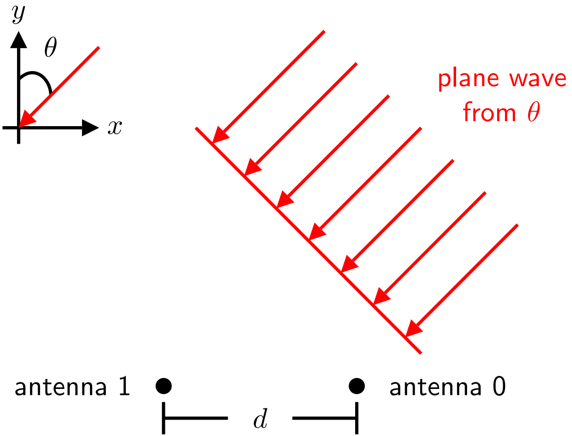

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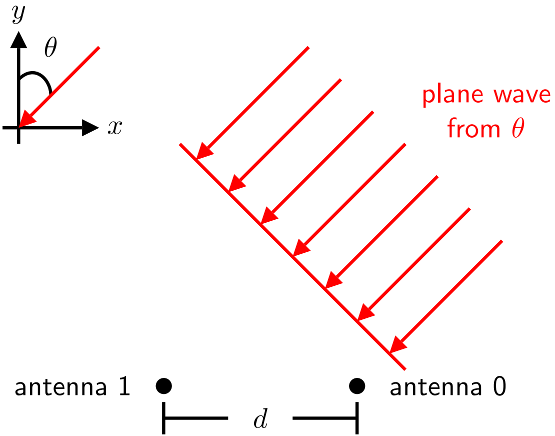

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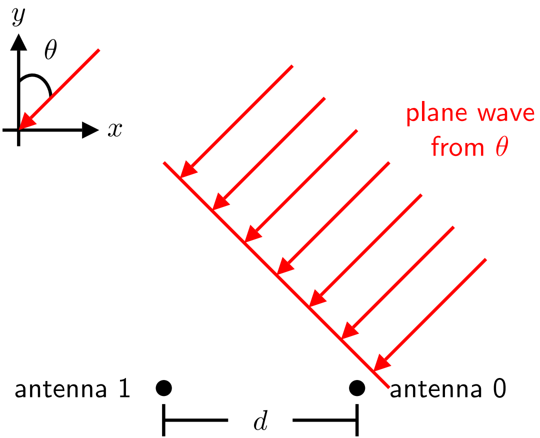

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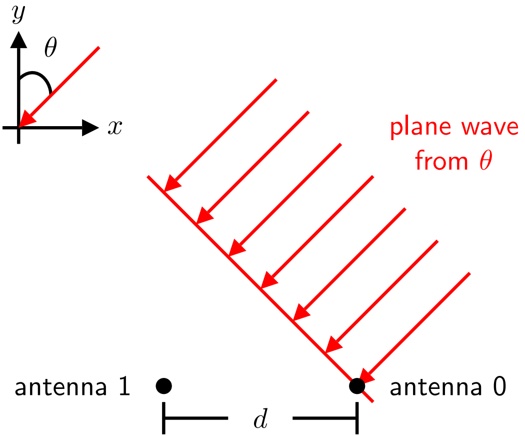

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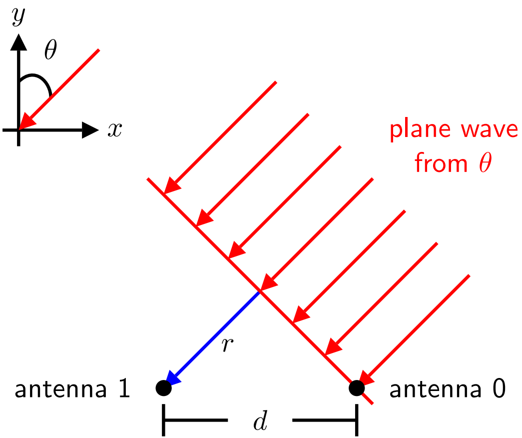

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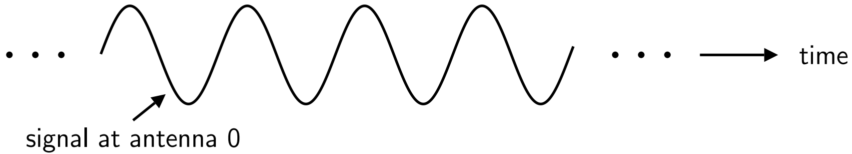
Signals reaching each antenna travel slightly different distances.

This extra propagation distance leads to a signal that is slightly delayed.



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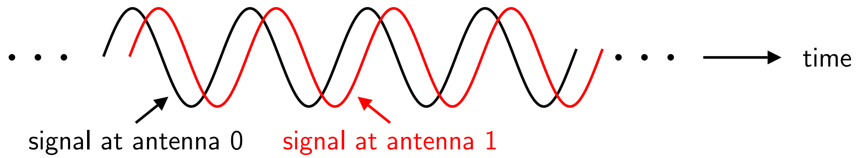

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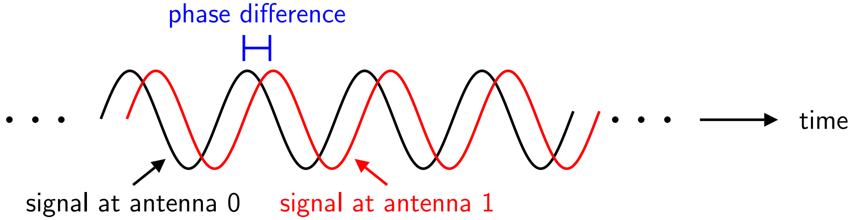

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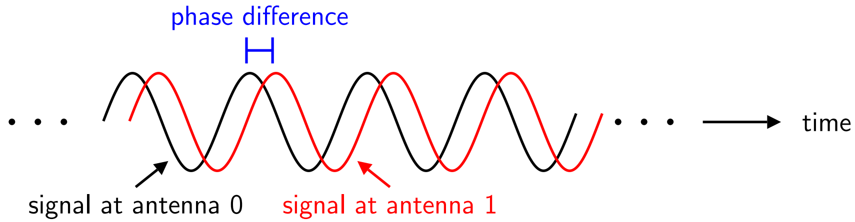
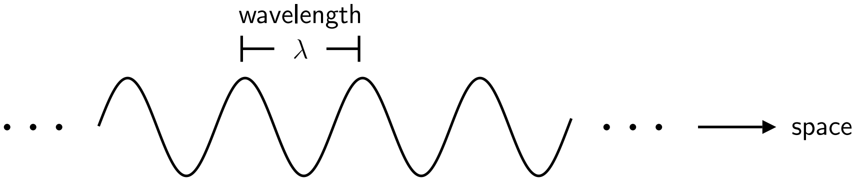


Let's look at how we can quantify this phase difference.

A wave completes one cycle (2π radians) after propagating λ meters.



$y(t) = \sin(2\pi f t)$

a sinusoidal wave

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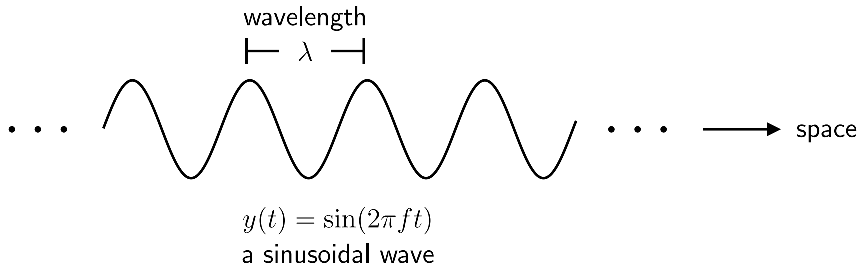

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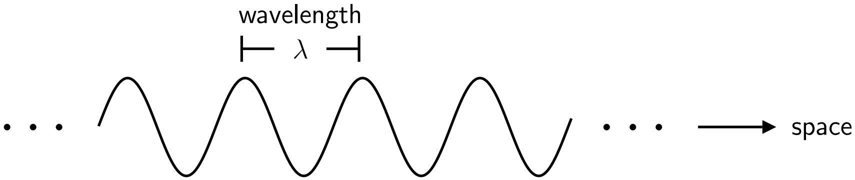

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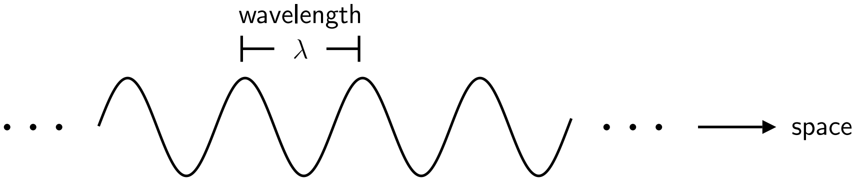

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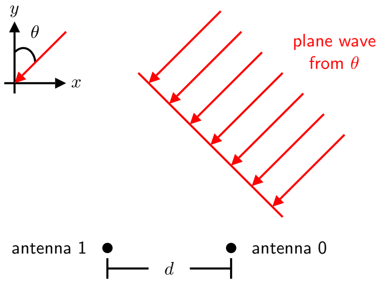

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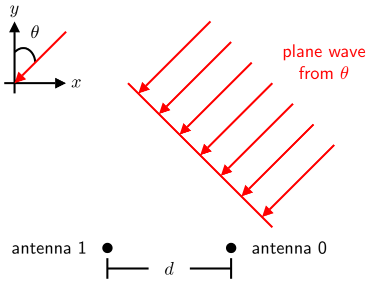

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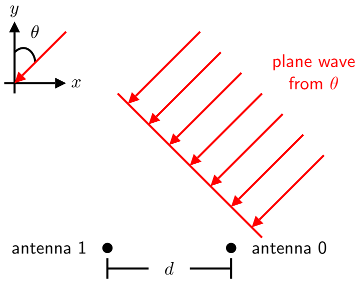

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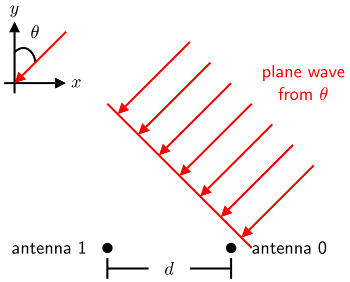

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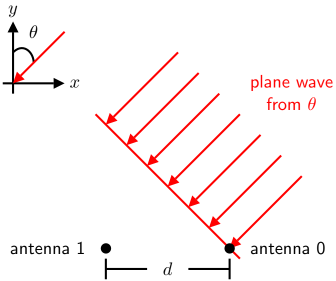

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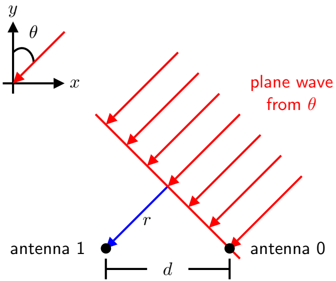

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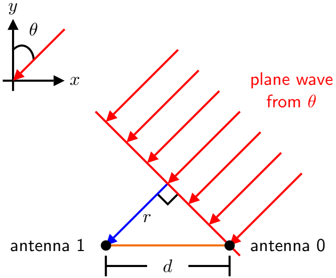

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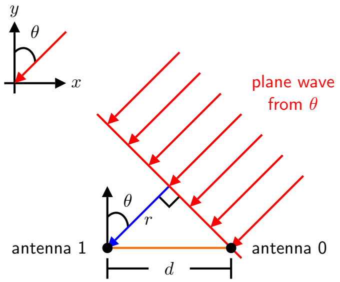

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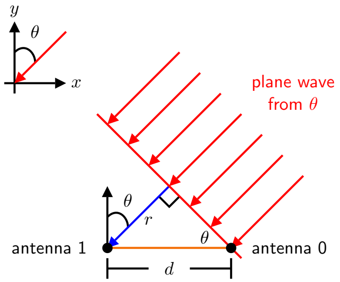

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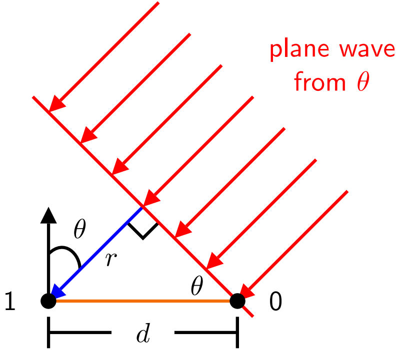

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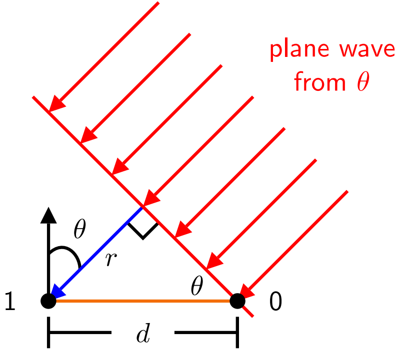

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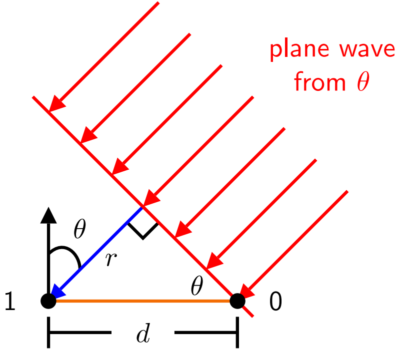

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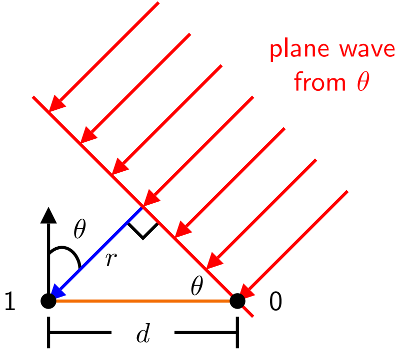

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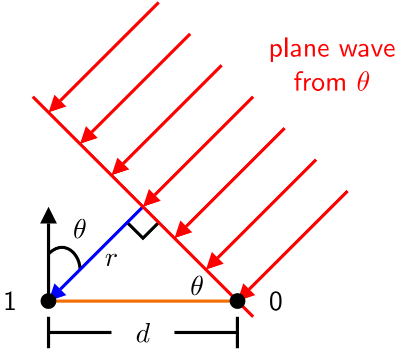
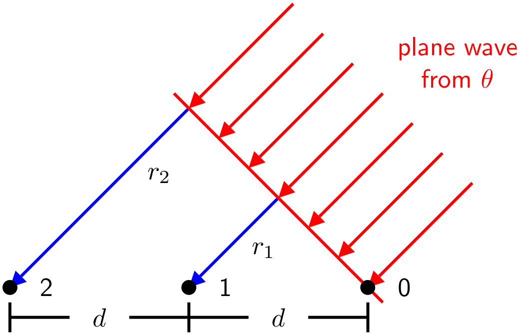
When $\theta < 0$, antenna 1 sees the signal ahead of antenna 0.



What if we have a third antenna?

In this 3-element array, we have

$$r_1 = d \sin\theta, \qquad r_2 = 2 \cdot d \sin\theta.$$



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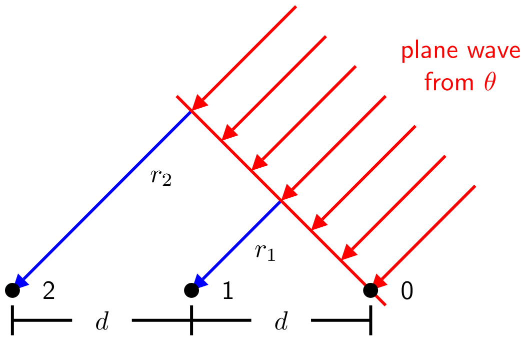

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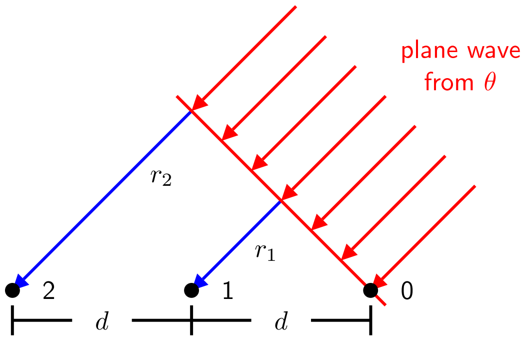
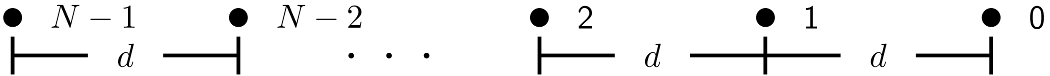


We can generalize this to an N-element linear array with uniform spacing d.



The signal at the i-th antenna can be written as

$$y_{i}(t) = y_0(t) \cdot \underbrace{\exp\left(-\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot i \cdot d \sin\theta\right)}_{\text{phase shift at antenna } i}. \tag{5}$$

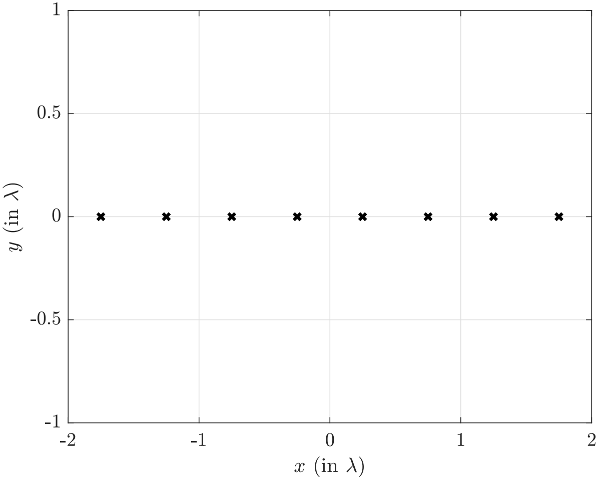
We can denote the phase shift at the i-th antenna induced by a plane wave from θ as

$$a_i(\theta) \triangleq \exp\left(-\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot i \cdot d \sin\theta\right). \tag{6}$$

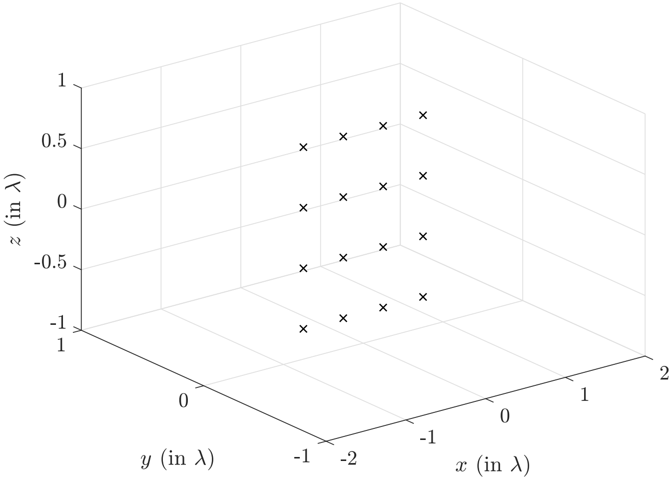
Collecting these phase shifts into a vector populates its array response vector.

$$\mathbf{a}(\theta) \triangleq \begin{bmatrix} a_0(\theta) \\ a_1(\theta) \\ a_2(\theta) \\ \vdots \\ a_{N-1}(\theta) \end{bmatrix} = \begin{bmatrix} \exp\left(-\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot 0 \cdot d \sin\theta\right) \\ \exp\left(-\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot 1 \cdot d \sin\theta\right) \\ \exp\left(-\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot 2 \cdot d \sin\theta\right) \\ \vdots \\ \exp\left(-\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot (N-1) \cdot d \sin\theta\right) \end{bmatrix} \in \mathbb{C}^{N \times 1} \tag{7}$$

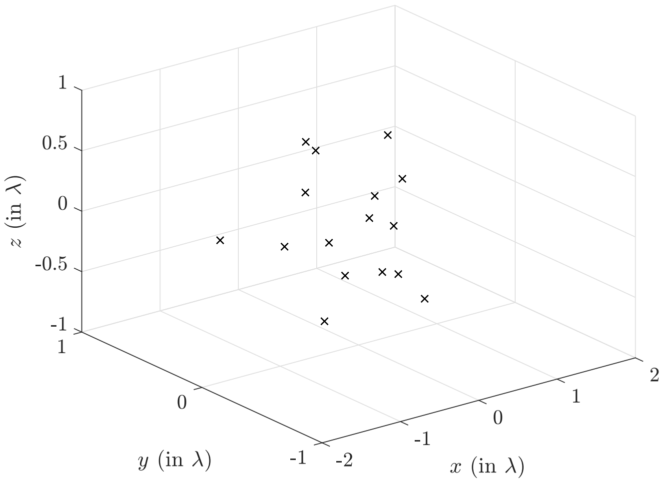
So far, we've only looked at uniform linear arrays in 2-D space.



What about uniform planar arrays in 3-D space?



What about arbitrary arrays in 3-D space?



A unit vector in the direction (θ, ϕ) can be decomposed into Cartesian coordinates as

$$x = \sin\theta \cdot \cos\theta \tag{8}$$

$$y = \cos\theta \cdot \cos\phi \tag{9}$$

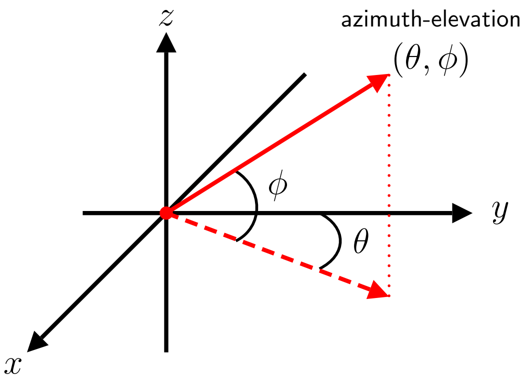
$$z = \sin\phi. \tag{10}$$

From Cartesian coordinates to azimuth and elevation, we have

$$\theta = \arctan\left(\frac{x}{y}\right) \tag{11}$$

$$\phi = \arctan\left(\frac{z}{\sqrt{x^2 + y^2}}\right). \tag{12}$$

Azimuth θ: left/right. Elevation ϕ: up/down.



Consider an array of N antennas, where the i-th antenna is located at (x_i, y_i, z_i) in 3-D space.

The relative phase shift experienced by the i-th antenna is

$$a_i(\theta, \phi) = \exp\left(\mathrm{j} \cdot \frac{2\pi}{\lambda} \cdot (x_i \sin\theta\cos\phi + y_i \cos\theta\cos\phi + z_i \sin\phi)\right). \tag{13}$$

The array response vector is then

$$\mathbf{a}(\theta, \phi) = \begin{bmatrix} a_0(\theta, \phi) \\ a_1(\theta, \phi) \\ \vdots \\ a_{N-1}(\theta, \phi) \end{bmatrix}. \tag{14}$$

This is the general form of the array response for any antenna array.

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- Changing one's coordinate system will change the expressions but the effective array response will not change.
- Applying a common phase shift to all elements will change the absolute array response, but practically we are only concerned with the relative phase difference across elements, which will be unaffected.

So...why does this router have multiple antennas? What does it do with them?

https://www.amazon.com/WiFi-6-Router-Gigabit-Wireless/dp/B08H8ZLKKK

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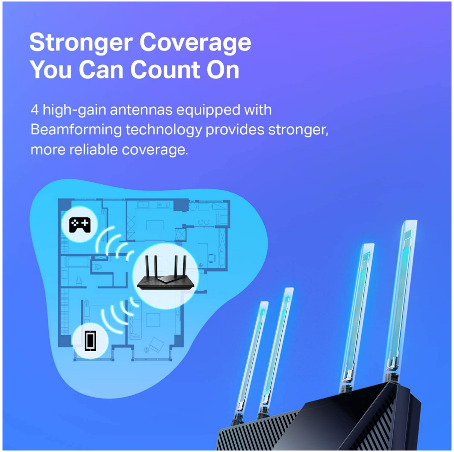

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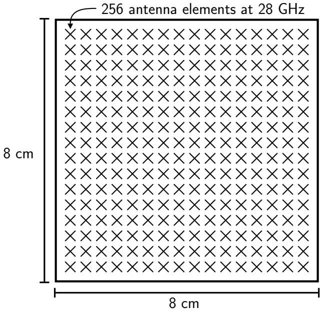
5G cellular systems also use multiple antennas, but often many more than Wi-Fi.



UE: user equipment, BS: base station.

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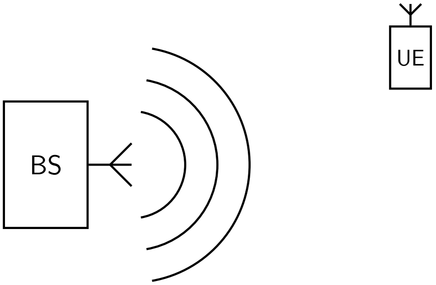

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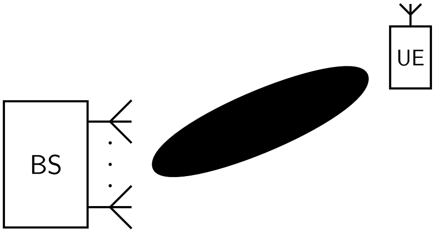

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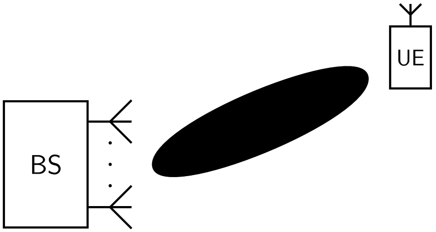
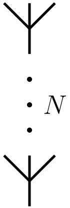


How to steer signals in a particular direction? → beamforming

UE: user equipment, BS: base station.

What is beamforming?



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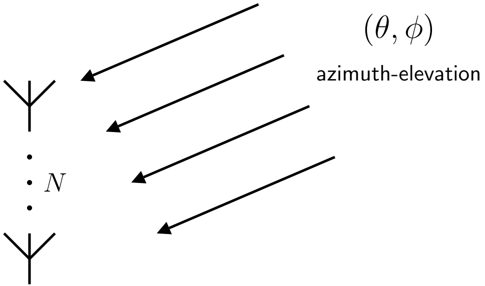

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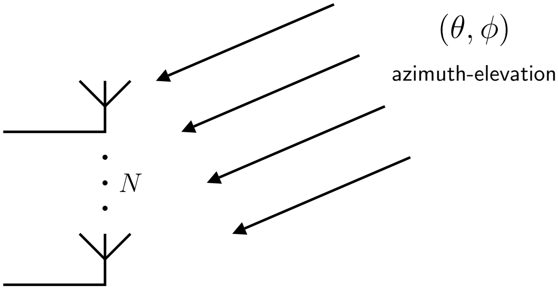

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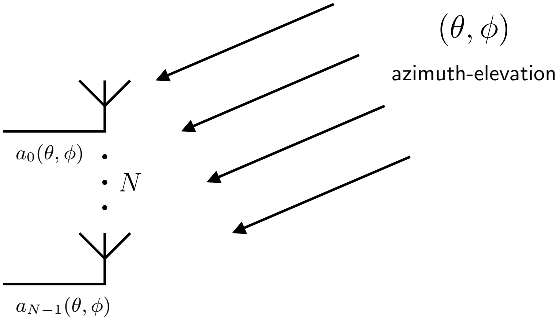

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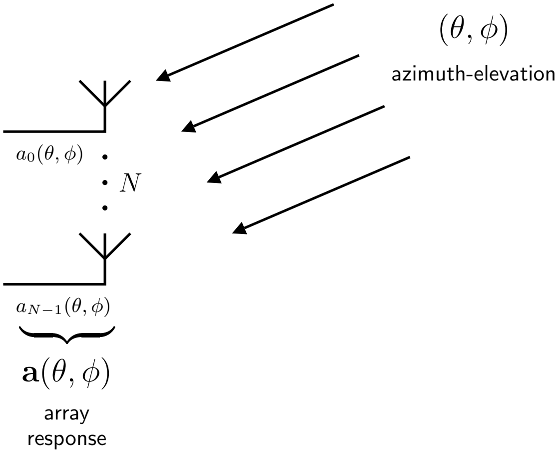

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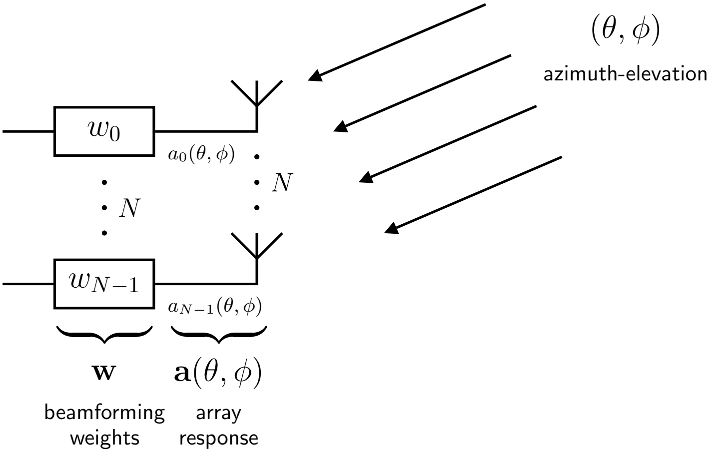

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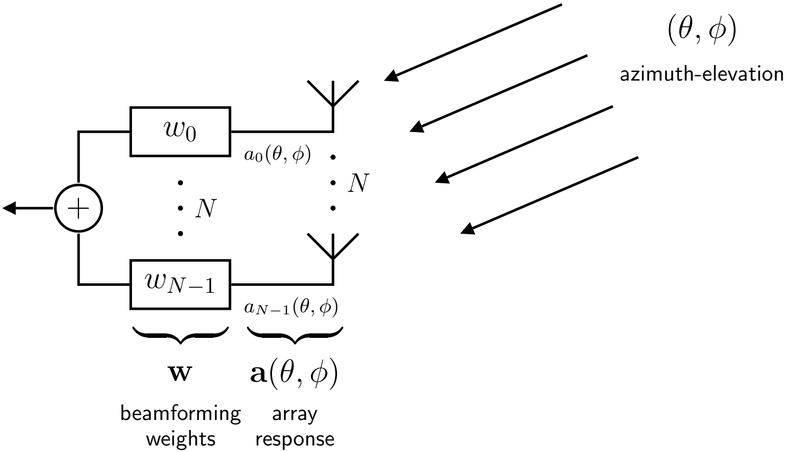

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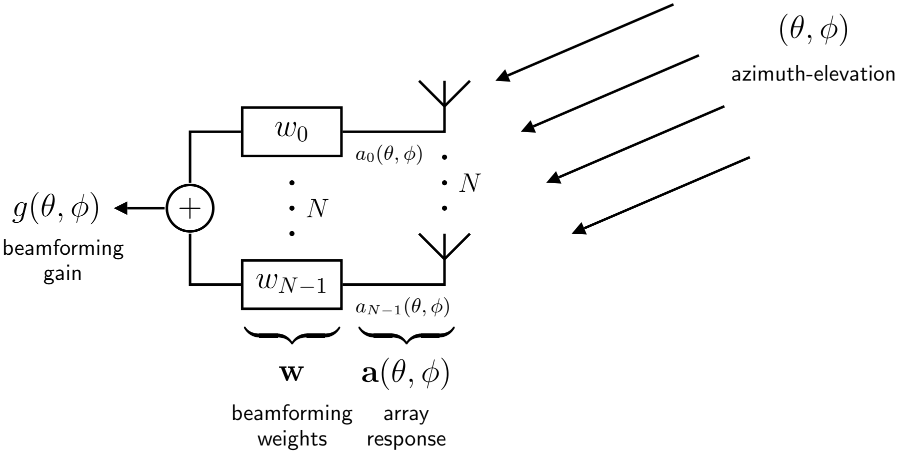
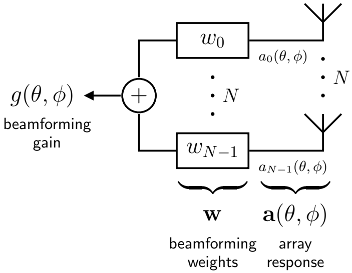


Let $y_i(t)$ be the signal striking the i-th antenna. The received signal after beamforming is

$$\sum_{i=0}^{N-1} w_i \cdot y_i(t) = \sum_{i=0}^{N-1} w_i \cdot \underbrace{a_i(\theta,\phi) \cdot y_0(t)}_{y_i(t)}$$



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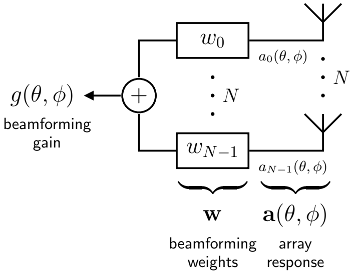

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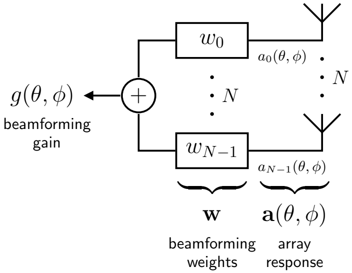

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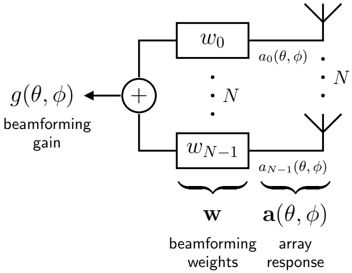

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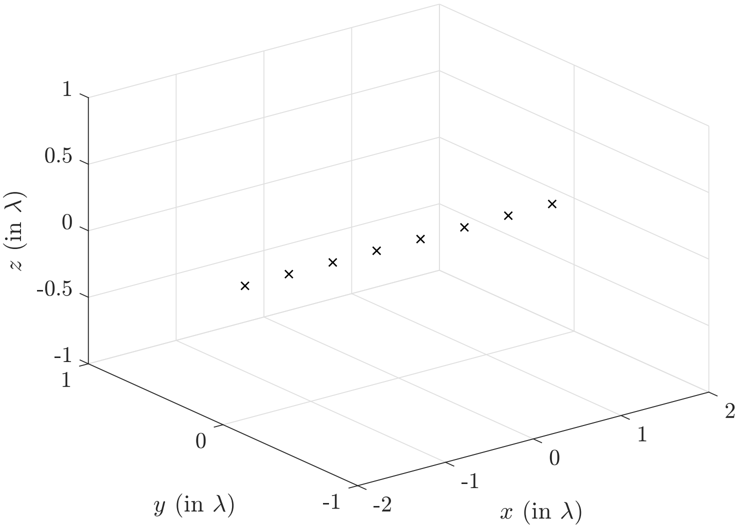
In communications, we're often interested in the magnitude of this gain, $|g(\theta,\phi)|$.
...but how do we design the beamforming weights to increase $|g(\theta,\phi)|$?

$$|g(\theta,\phi)| = \left|\mathbf{w}^{\mathrm{T}} \mathbf{a}(\theta,\phi)\right| = \left|\sum_{i=0}^{N-1} w_i \cdot a_i(\theta,\phi)\right|$$

Let's look at beamforming with an 8-element uniform linear array.



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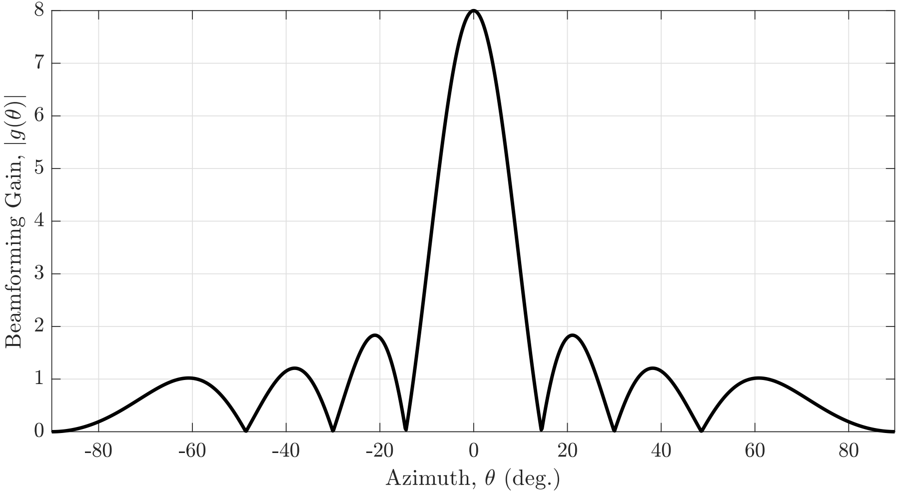

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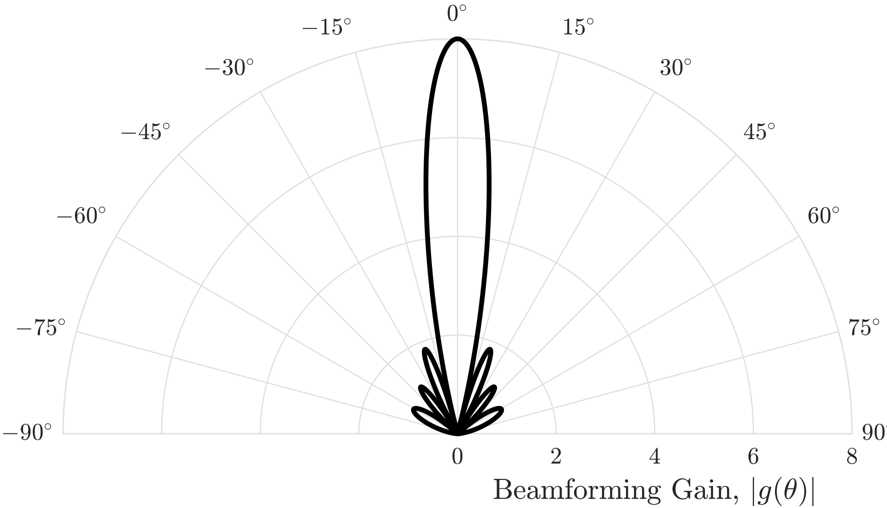
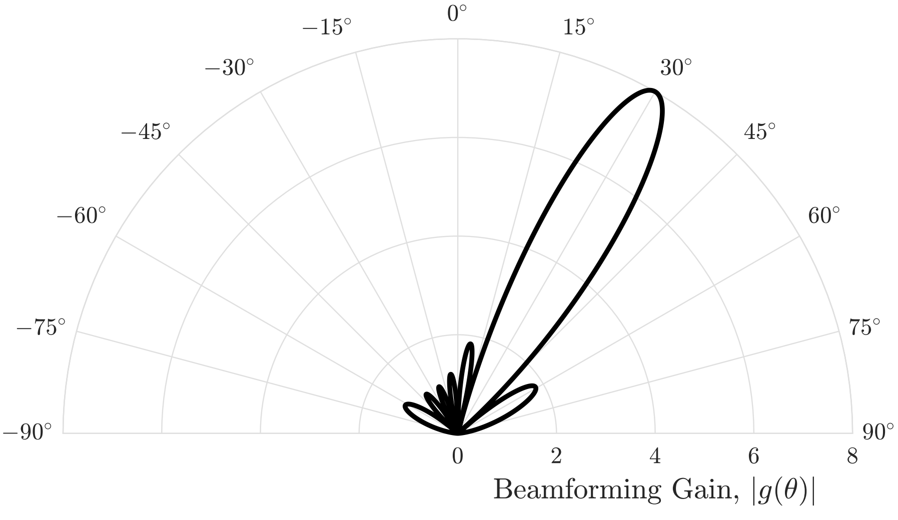
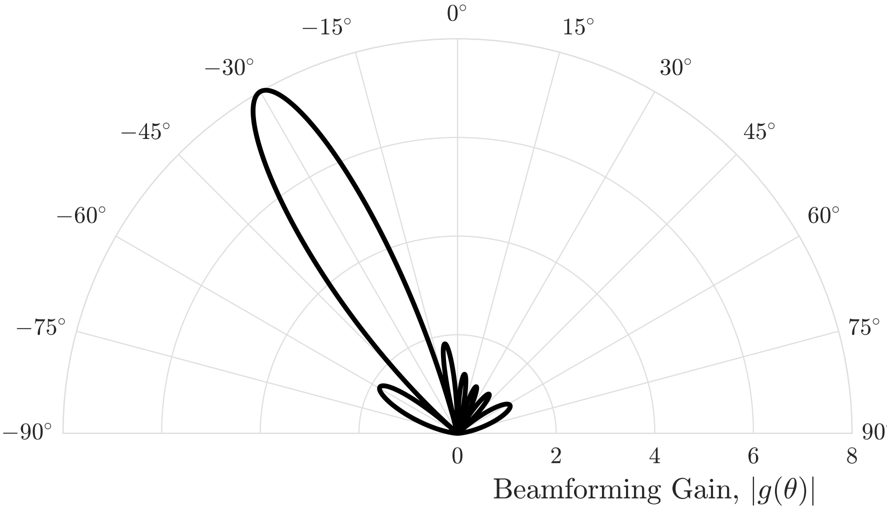


What if we choose $\mathbf{w} = \text{conj}(\mathbf{a}(30^\circ))$?



What if we choose $\mathbf{w} = \mathrm{conj}(\mathbf{a}(-30°))$?



To steer our beam toward some (θ, ϕ), we can use conjugate beamforming weights

$$\mathbf{w} = \mathrm{conj}(\mathbf{a}(\theta, \phi)) \iff w_i = a_i(\theta, \phi)^*. \tag{15}$$

Conjugate beamforming is also referred to as *matched filter beamforming*.

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This will "counteract" the phase shifts induced by the wave as it strikes the array.

$$y_i(t) = \underbrace{w_i}_{\text{weight}} \cdot \underbrace{a_i(\theta, \phi)}_{\text{response}} \cdot y_0(t) \tag{16}$$

$$= \underbrace{a_i(\theta, \phi)^*}_{\text{weight}} \cdot \underbrace{a_i(\theta, \phi)}_{\text{response}} \cdot y_0(t) \tag{17}$$

$$= \underbrace{|a_i(\theta, \phi)|^2}_{=1} \cdot y_0(t) \tag{18}$$

$$= y_0(t). \tag{19}$$

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The beamformed output signal under conjugate beamforming is then

$$\sum_{i=0}^{N-1} w_i \cdot y_i(t) = \sum_{i=0}^{N-1} w_i \cdot \underbrace{a_i(\theta,\phi) \cdot y_0(t)}_{y_i(t)} \tag{20}$$

$$= y_0(t) \cdot \sum_{i=0}^{N-1} w_i \cdot a_i(\theta,\phi) \tag{21}$$

$$= y_0(t) \cdot \sum_{i=0}^{N-1} a_i(\theta,\phi)^* \cdot a_i(\theta,\phi) \tag{22}$$

$$= y_0(t) \cdot \sum_{i=0}^{N-1} 1 \tag{23}$$

$$= y_0(t) \cdot N. \tag{24}$$

Conjugate beamforming with N antennas increases the signal in magnitude by a factor of N and in power by a factor of N^2, compared to receiving with a single antenna.

Thank you!

Please feel free to reach out to me with any questions at
`ipr@utexas.edu`